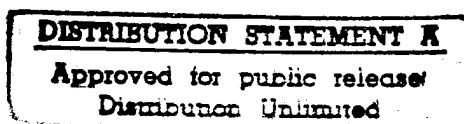
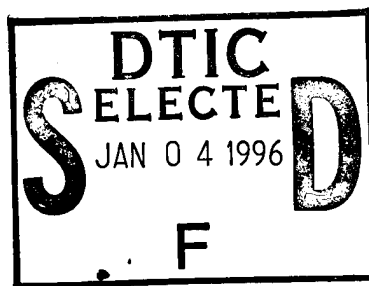


Creep and Strength of Frozen Soil Under Triaxial Compression

Anatoly M. Fish

December 1994



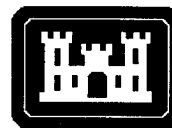
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Abstract

A combined creep and strength model has been developed for the entire (primary, secondary and tertiary) creep and the long-term strength of frozen soil under multiaxial stress at both constant stresses and constant strain rates by a single (unified) constitutive equation. Secondary creep is assumed to be an inflection point of a creep curve defining time to failure. Secondary creep rate is described by a new flow law, the stress function of which includes the first invariant of the stress tensor. The model consists of four principal elements: a constitutive equation, a viscous flow equation and a yield criterion, all united by a time-to-failure function. The yield criterion is selected either in the form of a parabolic (extended) von Mises-Drucker-Prager model or a parabolic (extended) Mohr-Coulomb rupture model. The criteria take into account that, at a certain magnitude of the mean normal stress (σ_{\max}), the shear strength of frozen soil reaches a maximum. The yield criteria are included in the time-to-failure function, the shape parameters of which are independent of the loading regime. The model has been verified using test data on creep and the long-term strength of frozen soil under triaxial compression at -10°C .

For conversion of SI units to non-SI units of measurement consult ASTM Standard E380-93a, *Standard Practice for Use of the International System of Units*, published by the American Society for Testing and Materials, 1916 Race St., Philadelphia, Pa. 19103.



**U.S. Army Corps
of Engineers**
Cold Regions Research &
Engineering Laboratory

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PREFACE

This report was prepared by Dr. Anatoly M. Fish, Research Civil Engineer, Civil and Geotechnical Engineering Research Division, Research and Engineering Directorate, U.S. Army Cold Regions Research and Engineering Laboratory. Funding was provided by the Office of the Chief of Engineers under DA Project 4A762784AT42, *Design, Construction and Operations Technology for Cold Regions*, Task BS, Work Unit D02, *Investigation and Design for Foundations in Cold Regions*.

Technical review was provided by F.D. Haynes (CRREL) and Dr. R.M.W. Frederking (Cold Regions Science and Technology). The author expresses special appreciation to F.H. Sayles, F.D. Haynes and E. Chamberlain for valuable comments.

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Creep and Strength of Frozen Soil Under Triaxial Compression

ANATOLY M. FISH

INTRODUCTION

It is well-known that the mechanical properties of frozen soil are similar to those of ice. Therefore, constitutive equations and strength criteria developed for one material are used interchangeably for the other. A considerable contribution in developing constitutive laws and failure criteria of frozen soils and ice under a complex stress-strain state was made by a number of researchers (Vyalov et al. 1963, Sayles 1973, Zaretsky and Vyalov 1971, Klein and Jessberger 1979, Vyalov and Slepak 1988, Domaschuk et al. 1991, Zaretsky 1993, Puswewala and Rajapakse 1993, and many others). A detailed consideration of the constitutive models of frozen soils and ice can be found in Puswewala and Rajapakse (1987). The basic assumptions on which these models are based can be divided into two groups. The models of the first group are based upon the following assumptions:

1. Creep and yield (strength) of frozen soil are two separate, loosely related phenomena described by different constitutive equations and failure criteria.

2. Creep of frozen soil is a three-stage process, comprising the primary, secondary and tertiary creep stages.

3. Failure of frozen soil takes place at the end of the secondary or the tertiary creep stages.

A three-dimensional combined creep and yield model of frozen soil presented below was originally developed by the author (Fish 1991b, 1992, 1993) to describe the rheological behavior of polycrystalline ice under multiaxial stress. The model is a generalization for a complex stress-strain state of a unified creep and strength model of frozen soil under uniaxial compression, developed earlier by the author (Fish 1980, 1983), based upon substantially different premises:

1. Creep and failure of frozen soils are a unified process that can be described by a single constitutive equation that includes a strength criterion of the material.

2. Short-term creep of frozen soil can be considered as a two-stage process, with the primary (decelerating) and the tertiary (accelerating) creep stages. Secondary creep is not considered a stage but only a (transition) point (M) on the creep curve (Fig. 1) defining time to failure.

3. The process of frozen soil failure begins at the start of deformation immediately after application of the load and takes place at all stages of creep.

Studies have shown that constitutive laws and failure criteria based upon the above premises led to more rigorous, complete and accurate constitutive laws and failure criteria for frozen soil and ice and to a successful solution of a variety of practical engineering problems in cold regions (Puswewala and Rajapakse 1993, and others). The present model consists of four principal elements: a constitutive equation, a viscous flow equation and a yield criterion, all united by a time-to-failure function. In the framework of this model, time to failure t_m performs a crucial role: it not only unites all stages of creep, which makes it possible to describe the entire creep process under both constant stresses and constant strain rates by means of a single constitutive equation, but it also combines the long-term strength and the yield criteria so that the latter can be incorporated into the constitutive equation of frozen soil.

CREEP MODEL

Constitutive equation ($\tau_i = \text{const.}$)

The entire deformation process, which includes primary, secondary and tertiary short-term creep of frozen soil under multiaxial stress at constant temperature, can be presented as a product of the flow equation and the nondimensional time function (Fish 1980, 1983, 1991b, 1993; Fish and Assur 1984)

$$\dot{\gamma}_i(\bar{t}) = \dot{\gamma}_{im} F(\bar{t}) \quad (1)$$

where

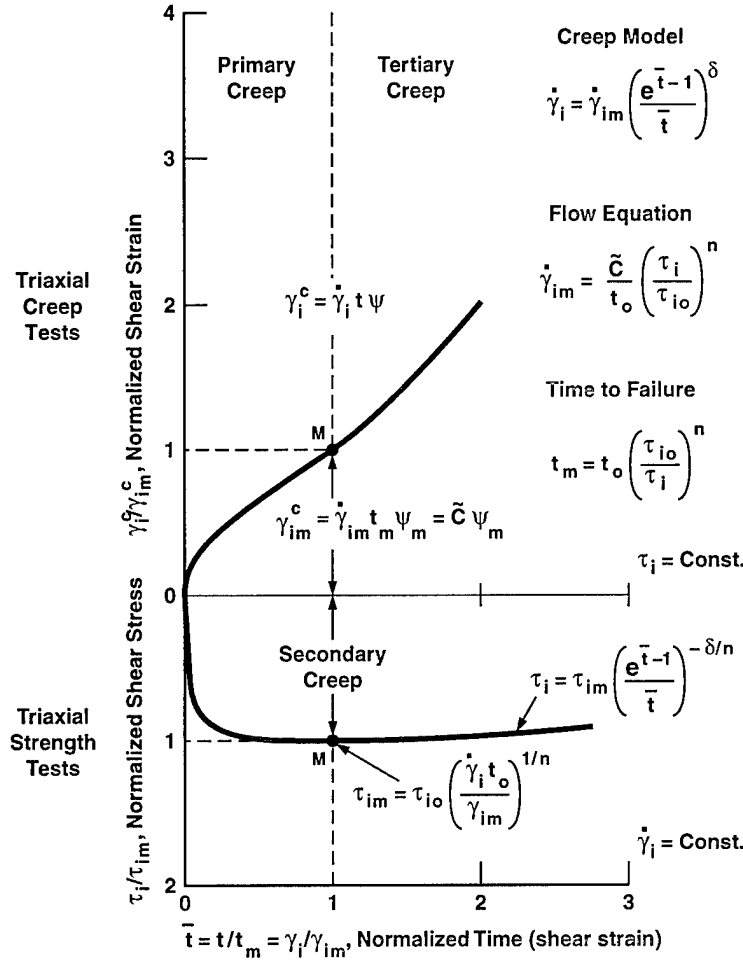


Figure 1. Creep model (after Fish 1983, 1991b).

$\dot{\gamma}_i(\bar{t}) = 2j_{2e}^{1/2} =$ shear strain rate (intensity)
 $\dot{\gamma}_{im}$ = minimum shear strain rate

$$j_{2e} = \frac{1}{6} [(\dot{\epsilon}_1 - \dot{\epsilon}_2)^2 + (\dot{\epsilon}_2 - \dot{\epsilon}_3)^2 + (\dot{\epsilon}_3 - \dot{\epsilon}_1)^2] \quad (2)$$

where

j_{2e} = second invariant of the deviatoric strain rate tensor

$\dot{\epsilon}_{1,2,3}$ = principal strain rates.

The nondimensional time function is

$$F(\bar{t}) = \left(\frac{e^{\bar{t}} - 1}{\bar{t}} \right)^\delta = \exp\{\delta(\bar{t} - \ln \bar{t} - 1)\} \quad (3)$$

where

$$\bar{t} = t/t_m \quad (4)$$

\bar{t} = normalized time

t_m = time to failure

δ = dimensionless parameter; $0 < \delta \leq 1$.

The function $F(\bar{t})$ is such that for primary and tertiary creep, $F(\bar{t}) > 1$, and for secondary creep

(points M in Fig. 1), $t = t_m$, $\bar{t} = 1$ and $F(\bar{t}) = 1$.

The parameter δ is the first (shape) parameter of a creep curve. For frozen soil δ is between 0.3 and 0.9, with an average value of $\sim 0.6 - 0.7$. The effect of changes of δ on the shape of creep curves is considered in Fish (1987, 1989).

It is essential that the creep process be considered not in terms of real time but in terms of normalized time, $\bar{t} = t/t_m$, where t_m is the time to failure (durability) of frozen soil, dependent upon the type of soil, its structure, the unfrozen water (ice) content, the temperature, the applied stress, the loading regime and other factors. Then, all the shape variations of the creep curves caused by the stress or temperature changes, or both, are expressed by variations of a single parameter, δ . In the certain stress domain, parameter δ may be assumed to be independent of stress (and temperature). Then, families of creep (and stress-strain) curves generated over a wide spectrum of mean normal stresses and

various temperatures can be superimposed on a single plot similar to that in Figure 1. One of the principal purposes of the author in developing this creep model (Fish 1980) was to describe the entire creep process at a constant stress and a constant strain rate under a simple, as well as a complex, stress-strain state by means of one parameter (δ) only.

Flow equation

The minimum shear strain rate is related to the time to failure by means of the flow equation (Fish 1980)

$$\dot{\gamma}_{im} = \frac{\tilde{C}}{t_m} \text{ or } \dot{\gamma}_{im} t_m = \tilde{C} \quad (5)$$

where \tilde{C} is the second shape parameter of a creep curve, which is equal to the viscous strain at failure. Both δ and \tilde{C} are assumed to be constant for a given soil temperature. The tilde (\sim) indicates that in the general case C may be a function of stress and temperature (Fish 1987, 1989). The magnitudes of \tilde{C} are between 0.01 and 0.3. A confirmation of validity of eq 5 for test data on the uniaxial compression of ice and frozen soil can be found in Mellor and Cole (1982, 1983), Fish (1983), Gonze et al. (1985) and others.

Note that the mathematical structure of eq 5, which is fundamental for this model, is reminiscent of the equation used by Garofalo (1965) and many others. However, eq 5 is substantially different from the latter. In Garofalo's equation, the product is assumed to be a constant value of the minimum strain rate by the time to rupture, $t_r \geq t_m$ (where t_r is the time of total soil failure at the end of the tertiary creep stage).

Time to failure

When time $t = t_m$ the strain rate reaches a minimum, $\dot{\gamma}_i = \dot{\gamma}_{im}$ ($\ddot{\gamma}_i = 0$), and it is assumed that failure occurs. The magnitude of shear stress that causes failure at time t_m can be expressed (Fish 1991b) as a product of two independent functions—the yield function (criterion) and the nondimensional time function $\Phi(\tilde{t})$

$$J_2 = J_{20} \Phi^2(\tilde{t}) \quad (6)$$

where

J_{20} = "instantaneous" yield criterion (Fig. 2)
 J_2 = second invariant of the deviatoric stress tensor.

The time function can be selected in the simplest form (Fish 1983):

$$\Phi(\tilde{t}) = (t_m / t_o)^{-1/n} ; \quad 1 \geq \Phi(\tilde{t}) \quad (7)$$

where

$\tilde{t} = t_m / t_o$ = nondimensional time to failure
 t_o = relaxation time (see eq 12 below)
 n = dimensionless parameter; $n \geq 1$.

By combining eq 6 and 7, the failure or the creep strength criterion of frozen soil takes the form

$$t_m = t_o \left(\frac{J_{20}}{J_2} \right)^{\frac{n}{2}} \quad (8)$$

or

$$t_m = t_o \left(\frac{\tau_{i0}}{\tau_i} \right)^n \quad (9)$$

where $\tau_i = J_2^{1/2}$ and $\tau_{i0} = J_{20}^{1/2}$ can be defined as the shear stress and the "instantaneous" shear stress intensity (resultant), respectively (see eq 23 and 26 below).

New flow law

Combining eq 5, 8 and 9 gives the flow equation of frozen soil in a multiaxial stress state in the form

$$\dot{\gamma}_{im} = \frac{\tilde{C}}{t_o} \left(\frac{J_2}{J_{20}} \right)^{\frac{n}{2}} = \frac{\tilde{C}}{t_o} \left(\frac{\tau_i}{\tau_{i0}} \right)^n \quad (10)$$

or

$$\dot{\gamma}_{im} = \tilde{C} \frac{kT}{h} e^{-E/RT} \left(\frac{\tau_i}{\tau_{i0}} \right)^n \quad (11)$$

in which

$$t_o \approx \frac{h}{kT} \exp \left(\frac{E}{RT} \right) \quad (12)$$

t_o \approx Frenkel's relaxation time (Frenkel 1947)

h = Planck's constant

k = Boltzmann's constant

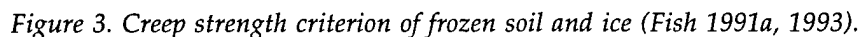
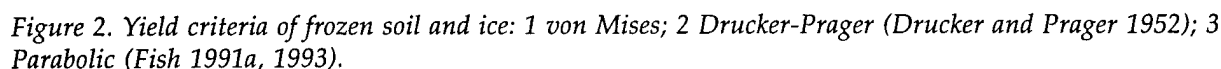
T = absolute temperature

E = activation energy

R = gas constant.

Both parameters n and $t_o(E)$ are strongly temperature-dependent. Temperature dependencies of these parameters are discussed in Fish (1985).

Note that the mathematical structure of the stress function is similar to that of the popular Norton-Glen power law used by many investigators (Vyalov et al. 1963, Sayles 1968 and others). Despite the fact that eq 10 and 11 contain a power function of stress, they are fundamentally different from the well-known Norton-Glen (1958) flow law. In contrast to the Norton-Glen equation, eq 10 and 11 contain parameters \tilde{C} and t_o , which have a definite physical



At a high stress level, when $\tau_i = \tau_{i0}$ and t_m

= t_o , eq 10 becomes

$$\dot{\gamma}_{io} = \frac{\bar{C}}{t_o} \quad (13)$$

where $\dot{\gamma}_{io} = \dot{\gamma}_{im}$ can be defined as the "instantaneous" shear strain rate, i.e., the rate at which the transition from the brittle to the ductile mode of failure takes place.

Creep shear strain

Creep shear strains are obtained by integrating eq 1. The entire creep process as described by Fish (1983) is

$$\gamma_i^c(t) = \frac{\tau_i}{G} + \dot{\gamma}_i t \psi \quad (14)$$

where

τ_i = shear stress

G = shear modulus

$\gamma_i^c = 2J_{2e}^{1/2}$ = creep shear strain

$$J_{2e} = \frac{1}{6}[(\epsilon_1 - \epsilon_2)^2 + (\epsilon_2 - \epsilon_3)^2 + (\epsilon_3 - \epsilon_1)^2] =$$

second invariant of the deviatoric strain tensor

$\epsilon_{1,2,3}$ = principal strains

$\psi = \psi(\bar{t})$ = integration coefficient calculated depending upon the normalized time.

Strain at failure

Failure occurs when $t = t_m$, $\bar{t} = 1$, and $\dot{\gamma}_i = \dot{\gamma}_{im}$. Equation 14 gives an expression for failure shear strain (Fig. 1)

$$\gamma_{im}^c = \frac{\tau_i}{G} + \dot{\gamma}_{im} t_m \psi_m \quad (15)$$

where ψ_m is an integration coefficient calculated for $\bar{t} = 1$. The first term in the right side of eq 15 is the instantaneous strain ignored in Figure 1.

Combining eq 5 and 15 gives

$$\gamma_{im}^c = \frac{\tau_i}{G} + \bar{C} \psi_m = \frac{\tau_i}{G} + \frac{\bar{C}}{\sqrt{\lambda}} \quad (16)$$

where $\psi_m \approx 1/(1-\delta)^{1/2} = \lambda^{-1/2}$ and $\lambda = 1-\delta$.

Failure strain criterion

For frozen soil the instantaneous strains are small and in many instances can be ignored, substantially simplifying the parameter evaluation procedure. Indeed, in this case eq 16 becomes

$$\gamma_{im}^c = \frac{\bar{C}}{\sqrt{\lambda}} \text{ or } \lambda = \left(\bar{C}/\gamma_{im}^c\right)^2 \quad (17)$$

It is interesting to note that eq 17 may be used as a strain failure criterion. Creep test data from the uniaxial compression of polycrystalline ice (Mellor and Cole 1982) showed that the magni-

tude of strain at failure (point M in Fig. 1) retains a constant value. On the other hand, test data on triaxial compression of frozen soil (Gorodetskii 1975) show that magnitudes of the failure strain vary with the change of confining pressure.

Creep strain calculations

Calculations of creep deformations are considerably simplified by using an approximate integral of eq 14 (Fish 1983 [eq 38], Fish 1987 [eq 20]):

$$\gamma_i^c(\bar{t}) \approx \frac{\tau_i}{G} + \frac{\bar{C}}{\sqrt{\lambda}} \bar{t}^\lambda e^{\delta_1(\bar{t}-1)} \quad (18)$$

where $\delta_1 = \lambda^{1/2} - \lambda$ and λ is also a shape parameter of a creep curve. When $t = t_m$, $\bar{t} = 1$ and eq 18 becomes eq 16.

Thus, the entire creep process of frozen soil in a multiaxial stress state at constant shear stresses is described by means of only two parameters, \bar{C} and $\delta(\lambda)$, to be determined from test data. An evaluation procedure for these parameters is presented in Fish (1983, 1987, 1989).

Triaxial constant strain rate tests ($\dot{\gamma}_i = \text{const}$)

The strength of frozen soil is determined from tests in which the shear strain rate is maintained constant ($\dot{\gamma}_i = \dot{\gamma}_{im}$); then the shear stress becomes a function of this rate (Fish 1983). Defining $\gamma_i = \dot{\gamma}_i t$, $\gamma_{im} = \dot{\gamma}_{im} t_m$, $\bar{C} = \gamma_{im}$ and combining the latter with eq 10 gives

$$\tau_{im} = \tau_{io} \left(\frac{\dot{\gamma}_i t_o}{\gamma_{im}} \right)^{1/n} \quad (19)$$

Subscript m indicates that the magnitudes of stress (τ_{im}) and strain (γ_{im}) are referred to the point M corresponding to the maximum (peak) shear stress in Figure 1. In the particular case when the applied strain rate $\dot{\gamma}_i = \dot{\gamma}_{im} = \dot{\gamma}_{io}$, then $\gamma_{im} = \gamma_{io} = \dot{\gamma}_{io} t_o$ and the peak shear stress $\tau_{im} = \tau_{io}$ is defined as the "instantaneous" yield stress (criterion) of frozen soil and ice and is given by eq 23 (26).

Introduction of the yield criterion τ_{io} and parameter t_o into eq 19 makes the latter essentially different from similar equations of Nadai (1950), Sayles and Epauchin (1966), Haynes et al. (1975), Ladanyi (1972), Baker (1979), and others. Correspondence between constant stress (CS) creep tests and constant strain rate (CSR) tests for uniaxial compression of ice and frozen soil is confirmed by the analysis of test data (Mellor and Cole 1983, Fish 1983 and others). The validity of eq 19 to test data on the strength of

polycrystalline ice under triaxial compression in a wide spectrum of strain rates is shown in Fish (1991b, 1992, 1993).

Stress-strain relationships outside point M are obtained by combining and rearranging eq 1, 3 and 19

$$\tau_i = \tau_{im} \left(\frac{e^{\bar{t}} - 1}{\bar{t}} \right)^{-\delta/n} \quad (20)$$

or

$$\tau_i = \tau_{im} \left(\frac{\gamma_i}{\gamma_{im}} \right)^{\delta/n} \exp \left[-\frac{\delta}{n} \left(\frac{\gamma_i}{\gamma_{im}} - 1 \right) \right] \quad (21)$$

where $t = t/t_m = \gamma_i / \gamma_{im}$.

For secondary creep, $\bar{t} = 1$ and the strain rate dependency of stress is defined by eq 19. The analysis of CSR test data showed (Fish 1983) that eq 21 gives a satisfactory approximation for the stress-strain relationships of frozen Fairbanks silt in uniaxial compression.

YIELD MODEL

Studies (Chamberlain et al. 1972, Sayles 1973, Gorodetskii 1975, Parameswaran and Jones 1981, Baker et al. 1982, Jones 1982, Jones and Parameswaran 1983) show that the shear strength of frozen soil and ice under triaxial compression is a nonlinear function of confining pressure. Moreover, at a certain magnitude of confining pressure defined (Chamberlain et al. 1972) as ice melting pressure, the shear strength of frozen soil (and ice) reaches a maximum value. A further increase of the pressure leads to a decrease of the frozen soil (ice) strength. On the other hand, in the small stress domain, a linear relationship is observed between the shear strength of frozen soil and the applied confining pressures.

A detailed consideration of yield criteria applied to frozen soil can be found in Vyalov et al. (1963, 1981), Vyalov (1986), Sayles (1973, 1988), Fish (1991a) and others.

In general terms a yield (strength) criterion for frozen soil (ice) under multiaxial stress at constant temperature that satisfies the above test data can be expressed in the form

$$\Phi_1(I_1, J_2) = 0 \quad (22)$$

where

I_1 = first invariant of the stress tensor

J_2 = second invariant of the stress deviator. Studies show that a parabolic yield criterion (Fig. 2) developed by Fish (1991a) well describes test data on the strength of frozen soil and ice over a wide range of confining pressures and various applied strain rates or times to failure, i.e.

$$\tau_i = c + b\sigma_m - \frac{b}{2\sigma_{max}} \sigma_m^2 \quad (23)$$

where

$$\tau_i = J_2^{1/2} = \frac{1}{\sqrt{6}} \left[(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2 \right]^{1/2}$$

= shear stress (intensity)

$$\sigma_m = \frac{I_1}{3} = \frac{1}{3} (\sigma_1 + \sigma_2 + \sigma_3) = \text{mean normal}$$

stress

$\sigma_1, \sigma_2, \sigma_3$ = principal stresses

c = resistance in pure shear associated with the frozen soil cohesion in the octahedral plane

$b = \tan \phi$, where ϕ = angle of internal friction in the octahedral plane

σ_{max} = ice melting pressure, i.e., the magnitude of the mean normal stress at which the shear stress of frozen soil reaches a maximum

$$\max \tau_i = c + \frac{b}{2} \sigma_{max} \quad (24)$$

The yield curve intersects the hydrostatic axis at points $H_{1,2}$, the abscissas of which are

$$h_{1,2} = \sigma_{max} \pm \left[\sigma_{max}^2 + \frac{2c}{b} \sigma_{max} \right]^{1/2} \quad (25)$$

Note that the yield curve shape is defined by the ratio of parameters c and b .

Equation 23 can be considered as an extended von Mises-Drucker-Prager yield criterion. At low stress levels ($\sigma_{max} \rightarrow \infty$), eq 23 transforms into the Drucker-Prager (1952) yield criterion. For frictionless materials ($b = 0$), eq 23 reduces to the von Mises yield criterion. Parameter σ_{max} may be regarded as one of the fundamental physical characteristics of frozen soil, probably closely related to the phase changing pressure (point H_1) for a given temperature (Nadreau and Michel 1986). Point H_2 with abscissa h_2 may be considered as the resistance of frozen soil to triaxial extension.

Equation 23 describes the strength of frozen soils in a multiaxial stress state, irrespective of

the time to failure or the applied strain rate by three temperature-dependent parameters— c , $b(\phi)$ and σ_{\max} —all of which have definite physical meanings and are easily determined from test data. It can be shown that a parabolic (extended) Coulomb-Mohr yield criterion (Fish 1991a) can also be applied to describe such a peculiar behavior of frozen soil under a high confining pressure.

CREEP STRENGTH

Creep strength criteria

It has been shown above that the creep strength of frozen soil is a function of time or applied strain rate. The absolute values of the frozen soil strength, as well as the magnitudes of the strength parameters, change from the greatest (instantaneous) values at the time of loading (t_0) to zero when $t_m \rightarrow \infty$. Consequently, the creep strength of frozen soil is characterized by a family of curves for given times $t_0 < t_1 < t_n$ or given strain rates $\dot{\gamma}_{10} > \dot{\gamma}_{11} > \dot{\gamma}_{in}$ (Fig. 3).

A *grapho-analytical* procedure for evaluation of the frozen soil strength based upon triaxial creep and strength tests data has been developed by Sayles (1973) using the Coulomb-Mohr rupture model.

An *analytical* procedure for interpretation of triaxial creep and constant strain rate test data and determination of strength parameters of frozen soil (and ice) has been developed by Fish (1991a, b, 1993, 1994) based upon the parabolic (extended) creep strength criteria.

It has been shown (Fish 1991a) that the creep strength of frozen soil can be expressed as a product of the parabolic yield criterion (eq 23) and a nondimensional time-to-failure function (eq 7), i.e.

$$\tau_i(t) = \tau_{i0}\Phi(\tilde{t}) = \left(c_0 + b_0\sigma_m - \frac{b_0}{2\sigma_{\max}}\sigma_m^2 \right) \Phi(\tilde{t}) \quad (26)$$

Equation 26 is a parabolic criterion for creep strength. When $t_m = t_0$, $\Phi(\tilde{t}) = 1$ and eq 26 becomes eq 23. The subscript 0 indicates that the failure shear stress or the magnitudes of the strength parameters refer to the instantaneous yield condition. Accordingly, the maximum shear strength is defined by

$$\max \tau_i(t) = \left(c_0 + \frac{b_0}{2}\sigma_{\max} \right) \Phi(\tilde{t}) \quad (27)$$

One may assume that the ratio of parameters b and c in eq 25 is constant, i.e., the positions of points H_1 and H_2 are unaffected by the change of the time to failure or the applied strain rate.

In the particular case in the range of small stresses when $\sigma_m \rightarrow 0$, the parabolic criterion eq 23 becomes the modified Drucker-Prager criterion for creep strength (Fish 1991, 1994)

$$\tau_i(t) = (c_0 + b_0\sigma_m)\Phi(\tilde{t}) \quad (28)$$

When $b = 0$, and eq 28 transforms into the modified von Mises yield criterion

$$\tau_i(t) = c_0\Phi(\tilde{t}) \quad (29)$$

Equations 26 through 29 are valid for constant strain rate (CSR) tests in which $\Phi(\tilde{t}) = \Phi(\dot{\gamma}_i)$, given by eq 31 and 32.

Creep strength criteria for frozen soil can also be selected in the form of a parabolic (extended) Coulomb-Mohr criteria, discussed in great detail in Fish (1991a).

Time-dependent strength

Passing vertical planes through point M in Figure 2, one obtains the relationship between the creep strength of frozen soil τ_i and the time to failure t_m . Such a dependency relates the instantaneous (yield) strength of frozen soil determined from creep tests at time t_0 (or from constant strain rate tests at the shear strain rate $\dot{\gamma}_{i0}$) with a creep strength $\tau_i(t) = \tau_i(t_m)$ determined at times $t_m = t_1, \dots, t_n$, etc. During creep at constant stresses, the shear strength of frozen soil as well as the strength parameters in eq 23 become functions of time (Fig. 3), i.e.

$$\tau_i(t) = c(t) + b(t)\sigma_m - \frac{b(t)}{2\sigma_{\max}}\sigma_m^2 \quad (30)$$

in which

$$\tau_i(t) = \tau_{i0}\Phi(\tilde{t}), \quad c(t) = c_0\Phi(\tilde{t}) \quad \text{and} \quad b(t) = b_0\Phi(\tilde{t})$$

Comparing eq 7, 9, 26 and 30 gives

$$\Phi(\tilde{t}) = \frac{\tau_i(t)}{\tau_{i0}} = \frac{b(t)}{b_0} = \frac{c(t)}{c_0} = (t_m/t_0)^{-1/n} \quad (31)$$

Consequently, for constant strain rate tests, time variations of the strength and the strength parameters in eq 26 through 31 are defined by the time function

$$\Phi(\dot{\epsilon}) = \Phi(\dot{\gamma}_i) = \frac{\tau_i(\dot{\gamma}_i)}{\tau_{i0}} = \frac{b(\dot{\gamma}_i)}{b_0} = \frac{c(\dot{\gamma}_i)}{c_0} = \left(\frac{\dot{\gamma}_{im}}{\dot{\gamma}_i t_0} \right)^{-1/n} \quad (32)$$

given by eq 19.

Note that the two parameters t_0 and n define the shapes of the curves of the long-term (creep) strength of frozen soil. It will be shown below for frozen soils at six different types of loading regimes (uniaxial compression and tension, pure shear, and triaxial compression at three different mean normal stresses) that, for all practical purposes, these parameters may be considered to be independent of the loading regime.

Thus, the entire process of deformation and time-dependent failure of frozen soil in a multiaxial stress state at both constant stresses and constant strain rates is described by seven parameters:

Creep	$\delta(\lambda)$ and \tilde{C}
Failure time	t_0 and n
Yield	c_0 , $b_0(\phi_0)$, and σ_{max}

which are determined from test data.

TEST DATA

Time-dependent strength of frozen soil

Test data on the time-dependent (creep) strength of frozen soil under various loading regimes used in the following analysis were

obtained by S.E. Gorodetskii (Vyalov et al. 1963, 1979, Gorodetskii 1975). Cylindrical samples of Kellovian sandy silt with a water content of 26%, a density of 1.81 g/cm³ and a specific gravity of 2.71 were used in the tests. The test procedures and the sample preparation technique are discussed in great detail in Vyalov et al. (1963, 1981).

Creep tests were carried out at six different loading regimes: uniaxial compression and tension, pure shear (torsion) and triaxial compression at three different magnitudes of the mean normal stress— $\sigma_m = 1.5, 3.0$ and 6.0 MPa. The tests were carried out at a temperature of $\theta = -10^\circ\text{C}$.

The magnitudes of shear stresses in the triaxial creep tests ($\sigma_2 = \sigma_3$)

$$\tau_i = (\sigma_1 - \sigma_3) / \sqrt{3}$$

were maintained constant by adjusting the magnitudes of the axial stresses in the cross section of the samples. In each series of tests, the magnitudes of the mean normal stresses

$$\sigma_m = (\sigma_1 + 2\sigma_3) / 3$$

were also maintained constant.

Test data on the long-term strength of Kellovian silt at -10°C under various loading regimes are presented in Figure 4. In these tests the time to

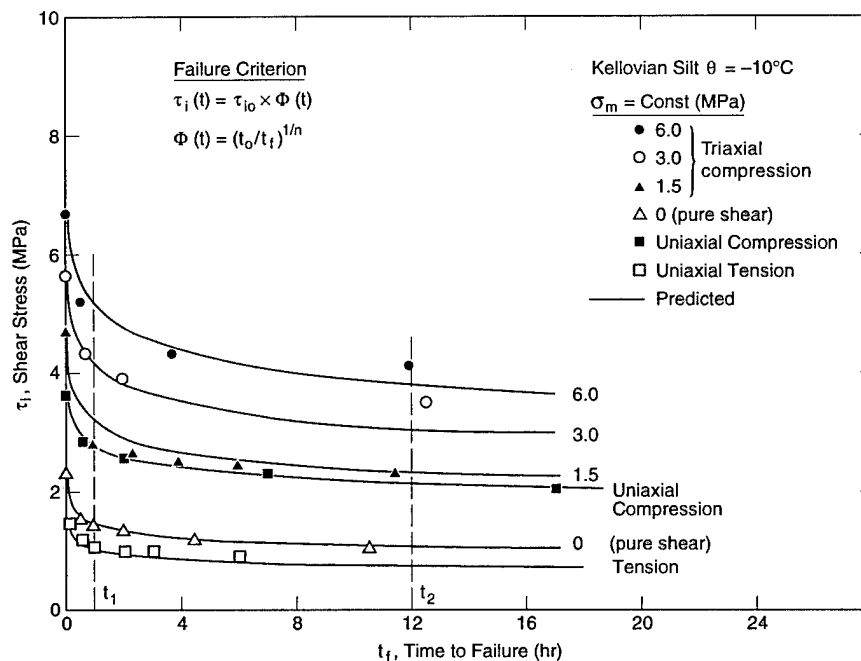


Figure 4. Creep strength of Kellovian silt under various loading regimes at -10°C . Data from Gorodetskii (1975) and Vyalov et al. (1963, 1981).

failure was calculated at the end of the secondary creep stage. Therefore, in the notation of t_f , the subscript f is used instead of m . It has been shown by Fish (1991a) that all the time dependencies of frozen soil strength presented in Figure 4 can be described by a single time-to-failure function (eq 9), two parameters of which

$$t_0 = 0.086 \text{ hr} \quad \text{and} \quad n = 9.64$$

are independent of the loading regime.

In Figure 5 the dependencies are presented of the shear strength upon the mean normal stress for different moments of failure time $t_f = 1$ hr and $t_f = 12$ hr. One can conclude that these dependencies can be described by the parabolic strength criterion eq 23. It has been found that the magnitudes of the strength parameters of Kellovian silt at -10°C are:

$$\begin{aligned} c_0 &= 1.98 \text{ MPa} & b_0 &= 1.66 (\phi_0 = 59^\circ) \\ \sigma_{\max} &= 6.38 \text{ MPa} & h_2 &= -1.1 \text{ MPa.} \end{aligned}$$

A detailed procedure for evaluation of these parameters is given in Fish (1991).

Figures 4 and 5 contain both test data and pre-

dicted dependencies of the creep strength of Kellovian sandy silt under various loading regimes and various moments of time. One can conclude that yield and creep strength criteria, eq 9 and 23–31, describe the test data well.

In Figure 6, time variations of the creep strength as well as the strength parameters of frozen soil are presented in the form of a dimensionless time-to-failure function calculated by eq 7. One can see that test points correlate well with the relationship predicted by eq 31.

From the above analysis, we can conclude that the shape parameters of the creep strength curves of frozen soil are independent of the loading regime, i.e., the magnitudes of the shape parameters t_0 and n do not vary with the change of the loading regime.

Creep of frozen soil under triaxial compression

In Figure 7 relationships are presented both for tests and for predicted creep curves of frozen Kellovian silt under triaxial compression ($\sigma_2 = \sigma_3$) at temperature -10°C . Test creep curves were obtained by S.E. Gorodetskii (Vyalov et al. 1979, Gorodetskii 1975) for various shear stresses: $\tau_i = 2.6, 2.29$ and 1.73 MPa. The predicted rela-

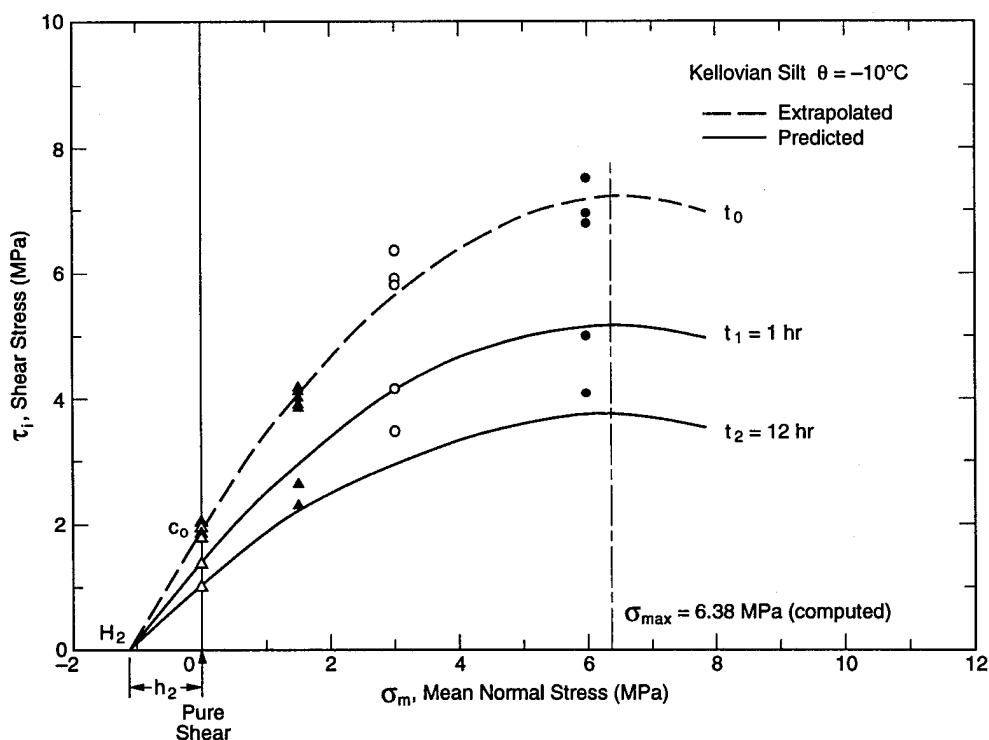


Figure 5. Shear strength τ_i vs σ_m , mean stress for Kellovian silt at -10°C . Data from Gorodetskii (1975).

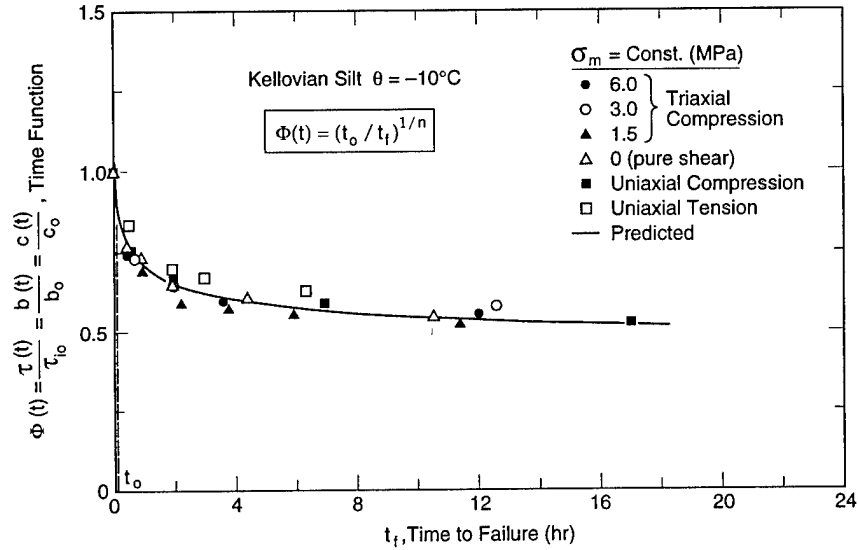


Figure 6. Normalized curve of the long-term strength of Kellovian silt at -10°C . Data from Gorodetskii (1975), Vyalov et al. (1963, 1981).

tionships were calculated by eq 17 and 18 using magnitudes of creep parameters: $\lambda = 0.2$ and $C = 0.16$.

The magnitudes of the times to failure t_f necessary for these predictions were calculated by eq 9 and by using the strength parameters of Kellovian silt at -10°C obtained above.

One can conclude that calculated dependencies of shear strain as a function of time correspond to test data on the creep of frozen soil under

triaxial compression and can be used for solving practical problems in cold regions.

CONCLUSIONS

1. The combined strength and creep model developed above describes the entire (primary, secondary and tertiary) creep and the long-term strength of frozen soil under triaxial compression.

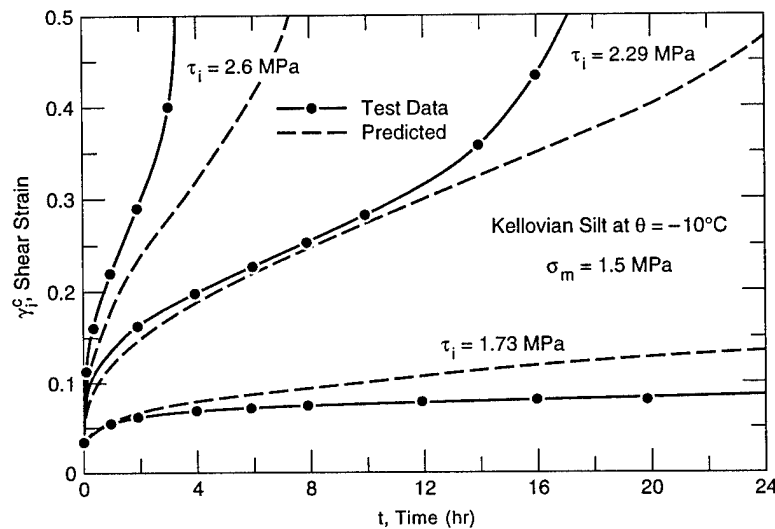


Figure 7. Creep curves of Kellovian silt under triaxial compression ($\sigma_2 = \sigma_3$) at -10°C . Data from Vyalov et al. (1979).

sion at both constant stresses and constant strain rates by a single (unified) constitutive equation.

2. The three-dimensional combined constitutive model consists of four principal elements: a constitutive equation, a viscous flow equation and a yield criterion, all united by a time-to-failure function.

3. The time-to-failure function or the equation of the long-term (creep) strength of frozen soil includes a parabolic criterion for creep strength, which is selected in the form of the modified (extended) von Mises-Drucker-Prager criterion or the modified (extended) Coulomb-Mohr yield criterion (Fish 1991a).

4. Time variations of the frozen soil strength as well as the strength parameters are described by the curve of the long-term (creep) strength, the shape parameters of which are independent of the loading regime and can be determined from tests under a sample stress-strain state such as uniaxial compression or pure shear.

5. The model has been verified using test data on the creep and time-dependent failure of Kellovian sandy silt at -10°C .

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REPORT DOCUMENTATION PAGE

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1. AGENCY USE ONLY (Leave blank)		2. REPORT DATE December 1994		3. REPORT TYPE AND DATES COVERED	
4. TITLE AND SUBTITLE Creep and Strength of Frozen Soil Under Triaxial Compression				5. FUNDING NUMBERS PR: 4A762784AT42 TA: BS WU: D02	
6. AUTHORS Anatoly M. Fish					
7. PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES) U.S. Army Cold Regions Research and Engineering Laboratory 72 Lyme Road Hanover, N.H. 03755-1290				8. PERFORMING ORGANIZATION REPORT NUMBER Special Report 94-32	
9. SPONSORING/MONITORING AGENCY NAME(S) AND ADDRESS(ES) Office of the Chief of Engineers Washington, DC 02314-1000				10. SPONSORING/MONITORING AGENCY REPORT NUMBER	
11. SUPPLEMENTARY NOTES					
12a. DISTRIBUTION/AVAILABILITY STATEMENT Approved for public release; distribution is unlimited. Available from NTIS, Springfield, Virginia 22161				12b. DISTRIBUTION CODE	
13. ABSTRACT (Maximum 200 words) A combined creep and strength model has been developed for the entire (primary, secondary and tertiary) creep and the long-term strength of frozen soil under multiaxial stress at both constant stresses and constant strain rates by a single (unified) constitutive equation. Secondary creep is assumed to be an inflection point of a creep curve defining time to failure. Secondary creep rate is described by a new flow law, the stress function of which includes the first invariant of the stress tensor. The model consists of four principal elements: a constitutive equation, a viscous flow equation and a yield criterion, all united by a time-to-failure function. The yield criterion is selected either in the form of a parabolic (extended) von Mises-Drucker-Prager model or a parabolic (extended) Mohr-Coulomb rupture model. The criteria take into account that, at a certain magnitude of the mean normal stress (σ_{max}), the shear strength of frozen soil reaches a maximum. The yield criteria are included in the time-to-failure function, the shape parameters of which are independent of the loading regime. The model has been verified using test data on creep and the long-term strength of frozen soil under triaxial compression at -10°C .					
14. SUBJECT TERMS Creep Failure Frozen soil Ice Model Strength Yield				15. NUMBER OF PAGES 20	
				16. PRICE CODE	
17. SECURITY CLASSIFICATION OF REPORT UNCLASSIFIED	18. SECURITY CLASSIFICATION OF THIS PAGE UNCLASSIFIED	19. SECURITY CLASSIFICATION OF ABSTRACT UNCLASSIFIED	20. LIMITATION OF ABSTRACT UL		